# AC Power

## Intro

### ILOs:

1. Define & calculate instantaneous power and average power (real power);
2. Calculate complex power, and use it to calculate average power, reactive power, and apparent power;
3. Determine power factors and explain whether and why they're leading vs. lagging; and
4. Design power-factor correction systems for inductive loads and explain why they're necessary.

Diagram

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### Topic 5 Videos:

1. Intro, analytical, & Multisim: <https://www.youtube.com/watch?v=Jx1cGY4e3-Q&list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&index=21>
2. Extra Problems:
   1. <https://www.youtube.com/watch?v=3AY-6zp7rq8&list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&index=22>
   2. <https://www.youtube.com/watch?v=Iy1nNjGDUJo&list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&index=24>
3. Lab Skills: <https://www.youtube.com/watch?v=Eibz4EBnXWI&list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&index=23>
4. Hantek Lab Modification: N/A (no new info needed for doing the lab with the Hantek this week)

### Deliverables for this week:

Note: As always, your full objective for this topic is to review these notes, the videos, practice problems, live class sessions and forum content, then to write-up creating and tri-solving a variation problem of the week's topic that demonstrates you've mastered the content. With that all still in mind, following are some specific guidelines & tips for this week.

See the Outline and Deliverable Rubric files for information for the write-ups in general. Specific tasks for your H5 deliverable are as follows:

1. Choose an input frequency for your supply to work at (one high enough that your inductors and resistors have the same order of magnitude of impedance)
2. Make a clearly-defined "load network" using at least one inductor and at least two resistors (remembering to include the series resistance of your "inductor"), placing components in parallel or series as you like, so that the power factor is < .9 and lagging (inductive load), and have a separate small "transmission line resistance" like in the sample lab. Calculate what the power factor is, showing your work.
3. Analytically find the capacitance that *would* correct the power factor of the load network, showing your work
4. You probably can't create this capacitance with the ceramic capacitors you have on hand, so go back and change your question (either your supply frequency, your load network, or both) until you can. Redefine this as your new load network power factor correction problem, and find the new power factor for this load network.
5. For the new problem, *demonstrate* the power factor correction by making your tri-solve objective to:
   1. Find the voltage and current (both amplitude and phase for each) and the complex power (both real and reactive power), both before and after adding the parallel capacitor to correct the power factor.
      1. Hint: position the small "transmission line resistance resistor" like in the sample lab so you can use it to measure the source current
   2. i.e., demonstrate that it worked for all 3 methods (analytical, multisim, and physical measurement), by calculating or measuring the source voltage and current phasors before and after adding the capacitor(s) and calculating the complex power.
6. Analytical: compare the three methods
7. Video: to help conclude the analog part of the course, make a 3-to-5 minute long video that briefly explains your problem & solution method and presents the results. *See the Deliverables Rubric file for a rubric & explanation*
8. Presentation: Your TA will have a 5 minute Q&A with you during the week of November 1-5 to give you a chance to answer questions and show your mastery of understanding of the analog part of the course *See the Deliverables Rubric file for a rubric & explanation*

## AC Power

### Instantaneous and Average Power

**Instantaneous power** is the actual power being delivered to a circuit element at any point, and is (as before) the product of the [instantaneous] voltage across and current through it . This is the power used by the element if we followed the passive sign convention in defining current direction relative to voltage change direction; i.e., defined  as the voltage change *from* where current  *exits* the element *to* where it *enters* it:

A picture containing diagram

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A circuit involving any combination of inductors, capacitors, and resistors powered by a single sinusoidal source (or even multiple sinusoidal sources that all have the same frequency) will have a sinusoidal current through and voltage across any element in the circuit we well, both with the frequency of the source: . In this case, we can simplify the instantaneous power being delivered to the source as follows:



Then, using , we have



**Average power** is the average of instantaneous power over one complete cycle (or equivalently, any integer number of cycles):



Since the average of a sinusoid like  over an integer number of cycles is zero, and since  is a constant in time, we get:



The cosine here is called the **power factor, .**

*Phasor Domain:*

*Hey! Listen! You can also find average power without leaving the phasor domain; we'll do more with this in 5.1.2, but for now, let's explore how impedance is related to power factor:*

Suppose you've found the voltage phasor across the element , and the element has impedance *Z*. Then 

so , meaning the power factor of a load is uniquely determined by the load's impedance, and is equal to the cosine of that impedance's argument (i.e., )

If pf = 1:

…the current and voltage are *in phase* and the element is constantly *dissipating* power (it's a resistive load).

If pf = 0:

…the current and voltage are 90o *out-of-phase* and the element is [on average] *not dissipating* power; it alternately stores and supplies it at different phases in a cycle

→ it's a purely capacitive or inductive load, so any energy given to it is not lost but stored either as potential energy either in the electric field via charge separation across a capacitor or in the magnetic field via current flow in an inductor.

In general:

…loads aren't either of these extremes and are complex (0 < pf < 1).

Furthermore, sources that *supply* power on average will have -1 < pf < 0 (power factor of -1 means it's constantly supplying power, like a lone DC source across a resistive load).

If  > 0 then the current *lags* the voltage (i.e., changes after it, as in an inductor), while if  then the current *leads* the voltage (i.e., changes before it, as in a capacitor).

Note: a sinusoid has the property that its rms value is its amplitude over sqrt(2): , so in terms of the RMS voltage & current  → avoiding the sqrt(2) factors is why people often prefer rms values when talking about AC power.

For a resistor, voltage and current are in phase, meaning , and  (pf = 1).

For an inductor, current *lags* the voltage by 90o, meaning  (**lagging** power factor = current lags the voltage), pf = 0, and  (not power consumed on-average).

For a capacitor, current leads the voltage by 90o, meaning  (**leading** power factor = current leads the voltage), pf = 0, and  (not power consumed on-average).

*Question: Suppose you find the power factor is +*0.5*, does this mean it's lagging or leading?*

Answer: impossible to tell. Power factor itself is  and cos is an even function, so the information about whether  or not is lost once you compute the numeric value.

→ Although we refer to power factors as leading or lagging, the numeric value of a power factor doesn't tell you which it is; for that you need to look at .

E.g., if  then pf = +0.707 (leading), meaning a capacitive load.

If , then pf = +0.707 (lagging), meaning an inductive load.

### Complex Power

We can also work with phasors the whole time and define terms that give the same results as in the previous section.

**Complex power** is  (here the  operator is the complex conjugate; if  then )



where

 is the **average power** dissipated (as before),

 is the **reactive power** (in volt-amperes, to distinguish it from actual power in watts), and

 is the **apparent power** (often in volt-amperes as well, to distinguish it from actual power in W, but quite often reported in W as well, e.g., in your electricity bill). Apparent power is greater than or equal to real power, and only equal to it if the power factor is 1.

→ Note that complex power is the product of voltage and [complex conjugate] current **rms** phasors, which for sinusoidal signals are . This is so that . In terms of regular (amplitude) phasors, we'd have .

Plotting the complex power in the complex plane gives a visual representation of it called the **power triangle:**

Diagram

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(the positive  shown indicates a *lagging* power factor (an inductive load). For a capacitive one the complex power phasor would point down).

### Power Factor Correction

Usually, you want electronic devices to have a power factor of 1. This is because the electricity company bills you for apparent power rather than actual power, so if you have a highly lagging power factor you'll be paying for much more energy than you're actually using. *It's not totally ridiculous for them to do this either; lagging power factor means that you're causing extra current flow that you're not actually using to power things but is still dissipating power in transmission lines, and you're messing with the phase and putting extra strain on the power grid compared to the same power with a unity power factor.*

**Power factor correction** means adding a capacitor in parallel with an inductive load (or an inductor in parallel with a capacitive one) to zero out the reactive power the *source* needs to supply without changing the voltage & current seen by the *load*. Ultimately, it works by letting the parallel capacitor supply the out-of-phase current to the inductive load so the source doesn't need to (or for the less-common case of correcting a capacitive load, it lets the parallel inductor supply the out-of-phase current to the capacitive load).

*Note:* power factor correctionto inductive loads are much more common because typically big electricity users are inductive either because they use coils to deliberately create magnetic fields (i.e., to step-down or step-up voltage with transformers, or to do mechanical work via a motor like in a vacuum cleaner, or compressors in air conditioners & refrigerators), or they use lots of wire (coiled up to save space) to create heat (like in an electric oven, water heater, or kettle), or both (like in a dryer or furnace).

*Question: What about a microwave oven?*

<https://www.youtube.com/watch?v=kp33ZprO0Ck&ab_channel=engineerguy>

#### Example Circuit

E.g., consider this circuit:

Diagram

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Suppose the "load" is the inductor in series with R1 (R2 represents the transmission line resistance, and is located there so we can use it to measure source current with the scope).

Ignoring R2, the current supplied will be , which will be fairly out-of-phase compared to the voltage:

Diagram

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The real power used by the load R1 is 304 μW, but the RMS current is 1.01 mA so the apparent power is more than twice the real power; *S* = Vrms*\*Irms* = 707 mV\*1.01 mA = 714 μW.

This RMS current means the source will be spending real additional power in the transmission lines, among other problems it can cause with electrical grids.

Note: the top numbers output by the probes are instantaneous values at the moment the simulation was stopped.

However, if we add a capacitor in parallel with the load (the L1 & R1 branch) we can reduce and phase-shift the current supplied by the source (removing the lag of the current) without affecting the situation seen by the load. To do this, we need the reactance of the load (the imaginary part of its impedance) to be 0.

Adding a capacitor in parallel will give an impedance of



and to make this purely real, we need



/\*\*\*Alternative derivation (ADMITTANCE):

Note that this calculation of how much capacitance to add in parallel is much easier if you work with admittances () rather than impedances, since admittances of elements in parallel just *add* ():

, therefore we need to add , and capacitive admittance is , therefore we need .

\*\*\*/

For *L* = 0.1 H, *f* = 1e3 Hz, and *R*1 = 300, we need . As promised, adding this capacitor lowers the current supplied by the source and phase-shifts it to line up with the voltage so that the source supplies the same real power and the LR load sees the same voltage & current, but we've reduced the power dissipated in the transmission lines and the apparent power the source needs to provide:

Diagram

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Specifically, while the real power to the load is still 304 uW, the transmission line losses have dropped from 10.1 uW to 1.88 uW, the rms current has dropped from 1.01 mA to 433 uA, and the apparent power (the one the electric company bills you on) has dropped from 714 uW to 306 uW. For a large factory with a monthly energy bill in the millions this is a very significant difference!

##### Why parallel and not series?

You might think we should remove the reactive part of the power by adding a capacitor in series instead of parallel. It's true that this could unify the power factor, but this method actually *increases* the power provided by the source and *drastically* increases the voltage seen by the load:

Diagram

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Although the reactive power the source provides is now zero (the current is phase shifted to line up with the voltage), but we've drastically increased the real power it provides by zeroing the reactive load this way. Because of this effect, you need to add the capacitor in parallel with the load if you don't want to change the voltage seen by the load.

Here's a few other ways to think about and understand this:

1. Because the source is still in parallel with the load after you add a capacitor in parallel, you haven't changed the voltage it sees and therefore haven't changed the current through it; just where that current comes from at which times (which is what we're trying to do). Adding it in series *can* change this though.
2. Thinking back to Topic 4, the frequency where an inductor and capacitor combination in series or parallel has no complex part to the impedance occurs is the *resonant frequency*! A resonant combination of inductor and capacitor in parallel has *higher* impedance (infinite without the resistor), so draws *less* current, but in series would have *lower* impedance (zero without the resistor) so draws *more* current!

##### Code to calculate this analytically:

Sample code calculating things in 5.1.3.1.

> **restart:**

**w:=2\*3.14159\*1e3: L:=.1: R1:=300: R2:=10:**

**ZLoad:=I\*w\*L+R1;**

**YLoad:=1/ZLoad;**

**YExtra:=Im(conjugate(YLoad));**

**C:=YExtra/w;**

**ZC:=1/(I\*w\*C);**

**ZNewLoad:=1/(1/ZLoad+1/ZC);**

**Vs:=1:**

**IOrig:=Vs/(ZLoad+R2); abs(%);**

**INew:=Vs/(ZNewLoad+R2);**

**VLoad:=IOrig\*ZLoad; abs(%);**

**VNewLoad:=INew\*ZNewLoad;**

**IC:=VNewLoad/ZC;**

**IL:=VNewLoad/ZLoad;**

**iC:=Re(IC\*exp(I\*w\*t));**

**iL:=Re(IL\*exp(I\*w\*t));**

**v:=Re(Vs\*exp(I\*w\*t));**

**plot([iC, iL, iL+iC, v/1e3], t=0..(4\*2\*Pi/w));**

#### Real-time Energy Generation in Ontario

See also: real-time energy generation in Ontario

<https://cns-snc.ca/media/ontarioelectricity/ontarioelectricity.html>